Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-08-27

Conceptual questions

Question 1. Draw the parametric curve $x = e^t, y = e^{2t}$. (When not stated, you should assume that the t interval is the largest possible, in this case from $-\infty$ to ∞.)

Question 2. If $x = f(t), y = g(t)$ is some parametric curve, how does $x = f(3t), y = g(3t)$ compare?

Question 3. Let C be the shape described by the equation $x^2 + y^2 = 5$. (What is this shape?) If we take C and

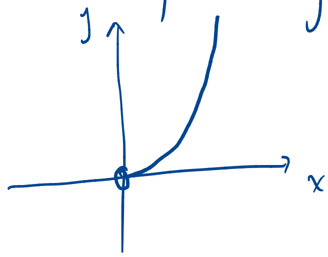
(1) shift it by 2 units in the positive y direction (i.e. upwards),
(2) and then stretch it by a factor of 3 in the x direction (i.e. horizontally),

what Cartesian equation describes the resulting shape?

Question 4. Come up with a parametrization $x = f(t), y = g(t)$ for the starting shape C in the preceding problem, and then a parametrization for the shape obtained after applying the transformations.

Question 5. Suppose a parametrization $x = f(t), y = g(t), a \le t \le b$ traces out a circle exactly once counterclockwise, ending where it started. One of the expressions $\int_a^b f(t)g'(t)\,dt$ and $\int_a^b g(t)f'(t)\,dt$ computes the area enclosed by the circle, and the other is its negative. Figure out which is which.

(1) Right half of parabola $y=x^2$, excluding origin



(2) Graphically it is the same curve, just traced 3 times as fast.

(3) C is a circle of radius $\sqrt{5}$ centered @ $(0,0)$.

move up: $x^2 + (y-2)^2 = 5$

then stretch: $\boxed{\left(\frac{x}{3}\right)^2 + (y-2)^2 = 5}$

(4) $x = \cos t, \quad y = \sin t \qquad 0 \le t < 2\pi$

parametrizes unit circle. So

$\frac{x}{\sqrt{5}} = \cos t \qquad \frac{y}{\sqrt{5}} = \sin t$

i.e.

$\boxed{x = \sqrt{5}\cos t \qquad y = \sqrt{5}\sin t}$

parametrizes C. Then do same transformations:

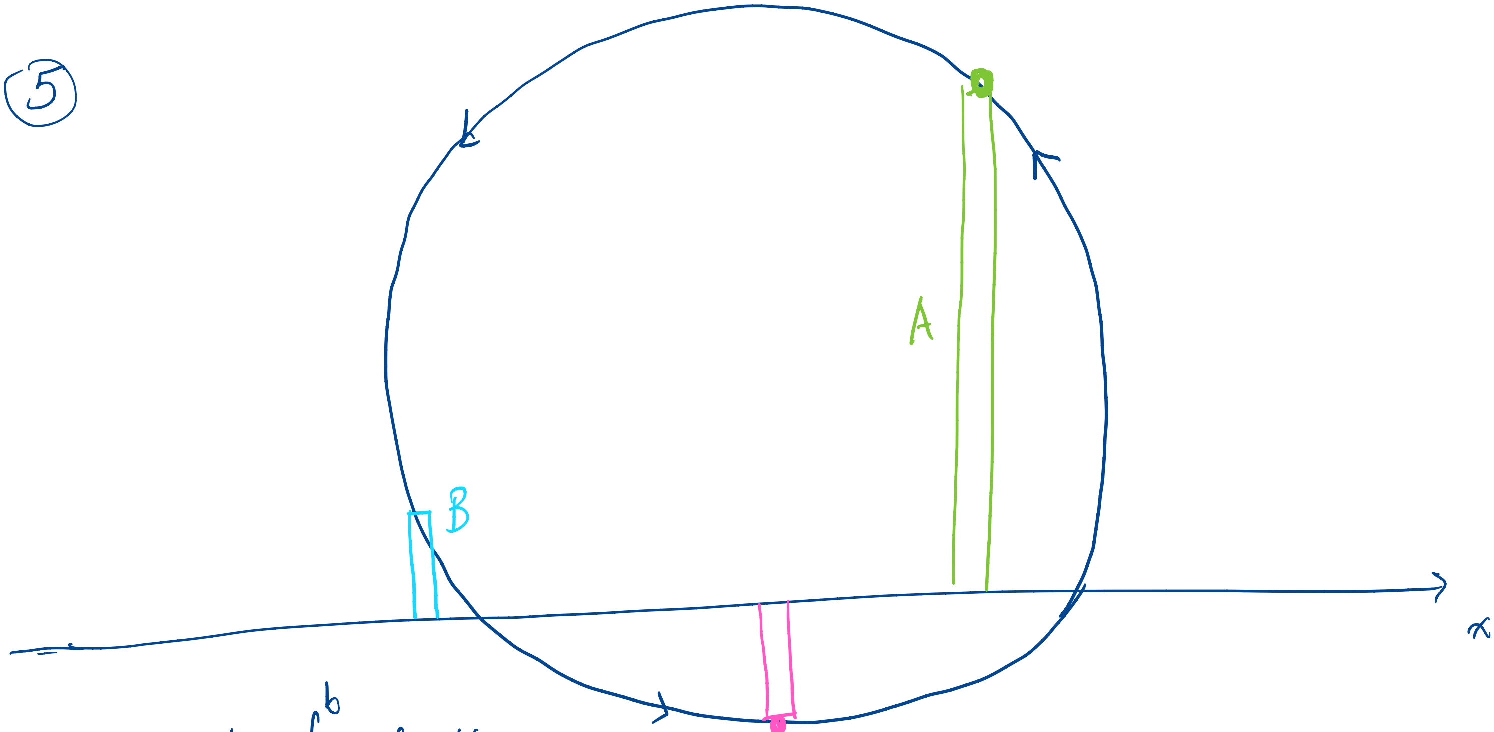
$\frac{x}{3} = \sqrt{5}\cos t \qquad y - 2 = \sqrt{5}\sin t$

i.e.

$\boxed{x = 3\sqrt{5}\cos t \qquad y = \sqrt{5}\sin t + 2}$

parametrizes the final ellipse.

5



Let's consider $\displaystyle\int_a^b \underbrace{g(t)}_{y}\,\underbrace{f'(t)\,dt}_{dx}$

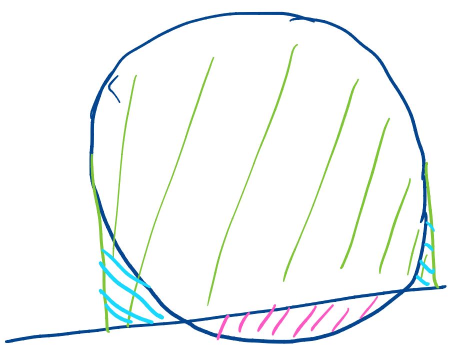
Think about the sign of $y\,dx$.

For A: $y>0$ $\quad dx<0$ $\quad -$

For B: $y>0$ $\quad dx>0$ $\quad +$

For C: $y<0$ $\quad dx<0$ $\quad -$

I get:



$-$  $+$  $-$  $= -$ (Area of circle).

(the integral $\displaystyle\int_a^b f(t)\,g'(t)\,dt$ gives the area, with correct sign.)